THE USE OF THE MERCATOR PROJECTIONS INTO THE CARTOGRAPHICAL REPRESENTATIONS

UTILIZAREA PROIECȚIEI MERCATOR ÎN REPREZENTĂRI CARTOGRAFICE

OCTAVIAN HERBEI *, ARTUR URSU **

**Universitatea din Petroşani*
***O.J.C.P.I. Timiş*

Abstract: *In general, the existent topographical maps of the Romanian territory are made by using on of the systems of cartographical projections: Stereographical projection – 1970, Gauss-Krüger or UTM (Universal Transversal Mercator). Each of these systems of cartographical projections has advantages and also disadvantages. One of the basic criteria in using a cartographical projection for a certain cadastral territory is that the relative linear deformation should be as small for that geographical area. The maps from the UTM projection have a great importance in the sea navigation and in the air navigation, because it is a conform projection, and the cartographical network is made from perpendicular lines, plain will be a right line. The same line makes with each of the meridians projection the same azimuth.*

Rezumat: *În general, hărțile topo – cadastrale existente în prezent pentru teritoriul României sunt întocmite folosind unul dintre sistemele de proiecție cartografică: Stereografica – 1970, Gauss - Krüger sau UTM (Universal Transversal Mercator). Fiecare dintre aceste sisteme de proiecție cartografică prezintă atât avantaje cat şi dezavantaje. Unul dintre criteriile de bază în adoptarea unei proiecții cartografice pentru un anumit teritoriu cadastral este ca deformația liniară relativă să fie cât mai mică pentru acea zonă geografică. Hărțile în proiecția UTM au o mare importanță în navigația maritimă şi aeriană, deoarece fiind o proiecție conformă, iar rețeaua cartografică formată din linii perpendiculare, loxodroma va fi o linie dreaptă. Aceeaşi linie face cu fiecare din proiecțiile meridianelor acelaşi azimut.*

Key words: *cartographical projections,* ***Gauss –*** *Krüger, Mercator, deformation*
Cuvinte chei:proiecții cartografice, Gauss – *Krüger, Mercator, deformații*

INTRODUCTION

The normal conform cylindrical projection has been studied and applied in 1569 by the Dutch cartographer Gerard Mercator, one of the main founders of mathematical geography. The projection studied by them is also called mercatorian projection.

In comparison with Gauss projection that consists in representing the globe on a developed surface of a cylinder whose declination ax is of 90° over the poles ax, the Mercator projection presumes to represent the globe on a developed surface of a cylinder whose declination ax is the same with the Earth rotation ax and tangent along the equator.

I. Gauss – Krüger Projection

a. Generalities

This projection was conceived in 1825-1830 by the German mathematician *Karl Friedrich Gauss*, and later, in 1912, *Johannes Krüger* elaborated the formulas necessary for changing the points coordinates from the rotation ellipsoid into the projection plan.

In Romania the Gauss projection was introduced in 1951, when it was adopted also the reference ellipsoid *Krasovski-1940*. The projection system was used for making the basic topographic plan at scale of 1:10.000, of the basic topographical map at scale of 1:25.000, and also of the unitary maps at different scales till 1973.

It is considered the rotation ellipsoid as a mathematical form of the earth and for the projection, the interior surface developed in plan of an imaginary cylinder, tangent to a meridian, which means in transversal position. (Fig.1).

For a unitary representation of the terrestrial ellipsoid into the projection plan there were established the meridians of tangency for whole globe, resulting a number of 60 geographical fuses each of 6° longitude, starting with original meridian called Greenwich.

For projecting the 60 fuses it is considered the ellipsoid to be covered in 60 successive cylinders, in horizontal position where each cylinder is tangent at the axial meridian correspondent to that fuse.

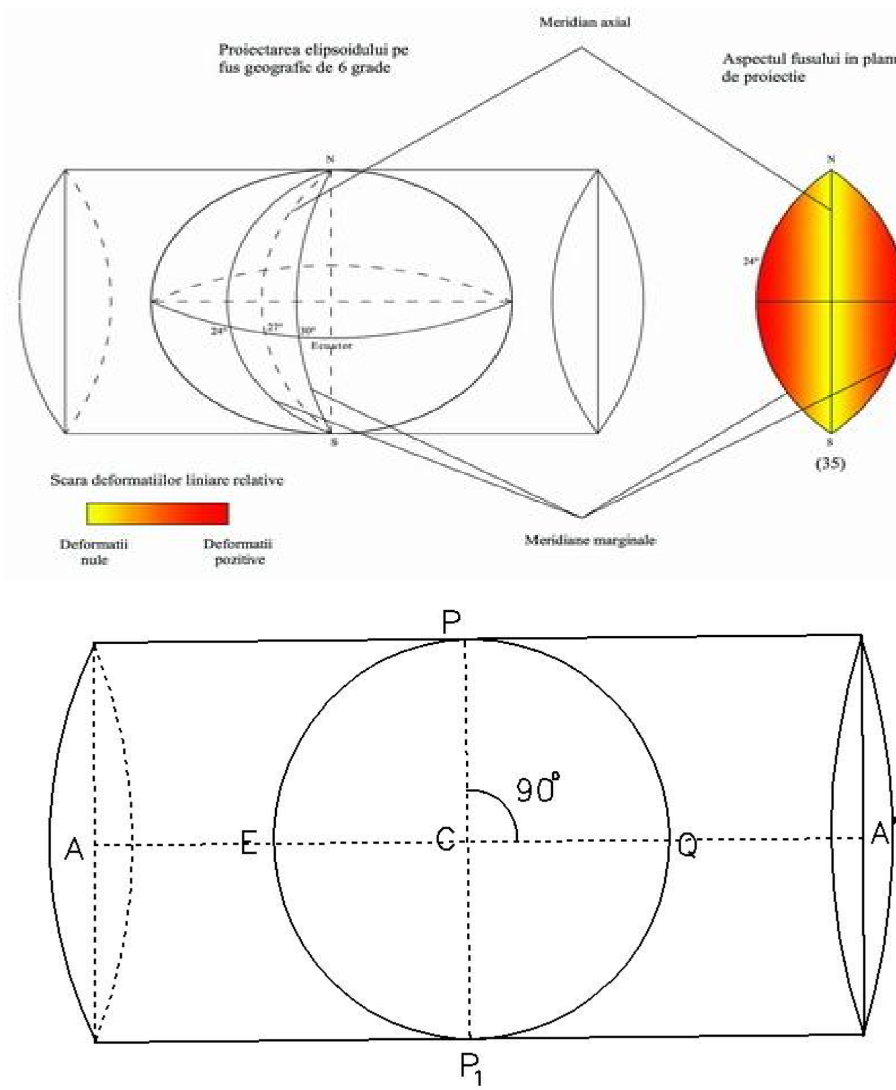

Fig. 1 –Gauss - Krüger projection

b) Deformations in Gauss – Krüger Projection

Relative linear deformation is expressed with the following formula**:**

$$D_{Gauss} = L^2 / 2R^2 + L^4 / 24R^4 + \ldots \qquad [km/km]$$

where:

- o D_{Gauss} is relative linear deformation in Gauss projection;
- o R is average ray of curve into the considered point;
- o $y=(y-y_0)$ is the distance from the point to the axial meridian.

It can be noticed from this formula and from this diagram that into the Gauss projection the relative linear deformations are positively and directly proportional to the axial meridian.

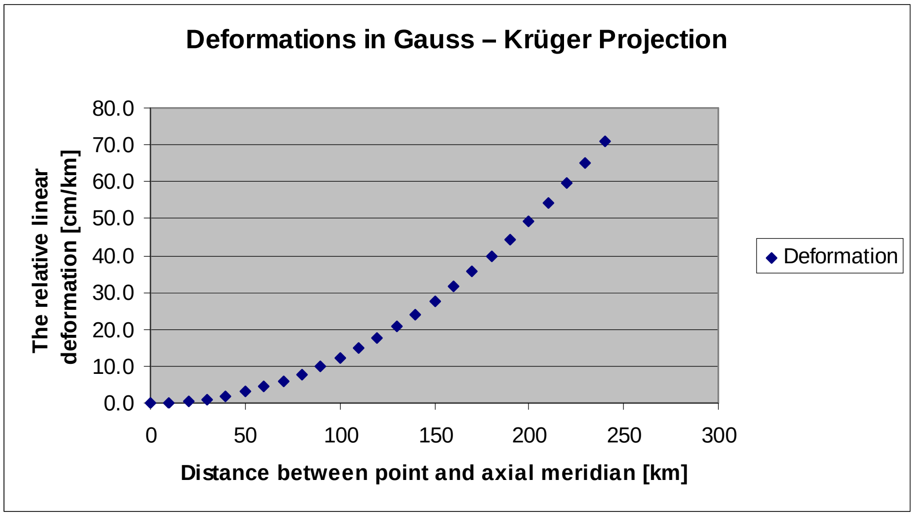

Fig. 2 The diagram of relative linear deformation into the Gauss projection

II. UTM Projection (Universal Transversal Mercator)

a. Generalities

This projection is a variant of the *Gauss – Krüger* projection, used in United States of America and in other countries, being important also for Romania due to our integration in new political and military structure.

The cartographical representation is made on fuses of 6° longitude, into the interval delimited by the parallels of 80° latitude south and 84° latitude north. The reference ellipsoid is the international ellipsoid called *WGS – 84*(Fig. 3), for which:

Great semi ax: $a = 6378137{,}000 \ m$

Geometrical flattening: $f = 1/298.257223563$

The projection cylinder (Fig. 4) is modified by reducing its dimensions and bringing it into secant with the ellipsoid along of 2 lines parallel with the central meridian. This means that in an area of 6 degrees there are 2 line of secant situated of almost 180 000 m E and V from the axial meridian. In order avoid to use the negative coordinates, to the central meridian it is attributed a false value of the east of 500 000 m E, this leading to the values of 320 000m E and respective 680 000 m V for the two secant lines.

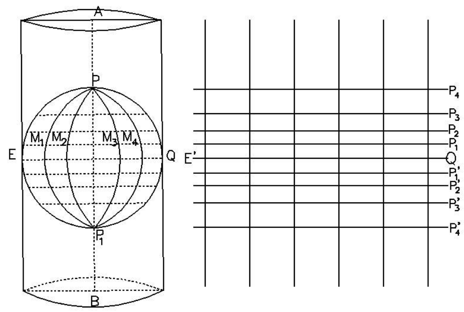

Figure 3 –UTM Projection

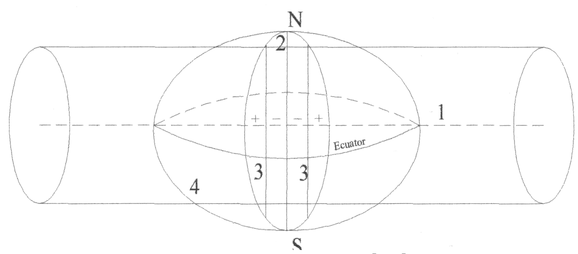

Figure 4 – Projection cylinder

1 – ax of the cylinder situated in equatorial plan;
2 – axial meridian;
3 – secant meridians;
4 – margin meridian of the fuse of 6 ;

- flexion ray of the meridian ellipse of φ latitude
- meridian arc β between 2 parallels φ_1 and φ_2

Adopting the representing system on fuses of 6° longitude, the plan representation is almost fiddled. Using the Mercator projection which is conform projection because the angles and the module u of linear deformation are not deformed very much it leads to a precise representation of the earth. It disadvantages are that the calculation is very difficult but due to the modern technology of calculation this is not observing.

b) Characteristics

- Projection: transversal cylindrical Mercator (= Gauss Kruger) in areas of 6 longitude
- Reference spheroid International World Geodesic System (WGS 84)
- Original longitude: central meridian of each area
- Original latitude: 0 degrees (Ecuador)
- Measuring unit: meter
- False north: 0 meters (10 000 000 m for south hemisphere)
- False east: 500 000 m
- Reduction coefficient: k_0 = 0,8996

- Areas numbering: it is started with the first area between 180 west and 174 west continuing to the east at 60 between 174° east and 180° east
- Limits of system latitude: North = 84° N, South = 80° S
- Areas limits: are delimited by the meridians whose longitudes are multiplied with 6 degrees east or west to the Greenwich meridian
- Systems of coordinates for each fuse.

c) Scale Factor

For certain special operation of geodesy and artillery where there are involved long distances and great precisions there are necessary some corrections for distances measured on the map. These can be determined by using the scale factor. For the transversal Mercator projection the scale factor is 1000 along the secant lines, decreasing till 0,9996long the axial meridian and increasing till 1,0010 into the limits of the fuse.

Conditions for projecting the ellipsoid on the plan:

- The representation must be conform (the angles should not be deformed)
- The representation of the axial meridian of a fuse is a straight line against to which the projection is symmetric
- The scale on the axial meridian direction is K_0 = 0,9996 which means that the cylinder that covers the ellipsoid is not tangent to the axial meridian but it is secant after 2 symmetrical meridians against the axial meridian.

The meridians and parallels are represented through some curves (Fig.5)

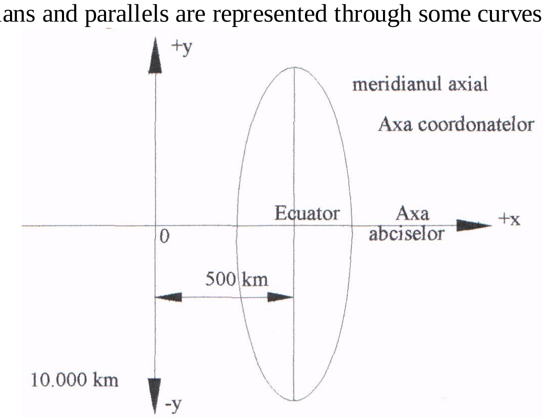

Figure 5 – System of axes in a fuse

The position of a point O into the projection plan is determined into a network of rectangular coordinates x and y. the ax y is the same with the meridian ax of the fuse and the ax x is the same with the parallel of the equator. In order to avoid having the negative abscises from the west against the meridian abscise x = 500 000 m. In order to avoid to have the negative coordinates from the south hemisphere it is adopted that the equator should have the coordinate y=10 000 000.

Justifying the value y=10 000 000

- from the equator to south pole is 90° latitude
- 1° = 111 km
- 90° * 111 km=9 990 000 m ~ 10 000 000 m

The relation which allows crossing from the Gauss coordinates (x,y) to plane coordinates UTM (N,E) is as follows:

(Distance from the UTM projection/ distance from the Gauss projection) = K_0 = 0,9996 = const.

$$N = xK_0$$

$$E = yK_0$$

And the scale (the linear deformation module) from the UTM projection can be calculated with the following relation:

$$m_{UTM} = K_0 m_{GK}$$

d) Maps Nomenclature in Mercator projection

As basic scales the NATO standards proposes the scales of 1:250 000 and 1:50 000. each scale has its own nomenclature.

- **Map sheet nomenclature at scale of 1:250 000**

Dividing the ellipsoid surface in fuses corresponds with the draft for the international map sheets where the Earth surface is covered with a series of geometrical figures whose dimensions are of 6° longitude and 4° latitude. The fuses of 6° longitude in Mercator projection are numbered from 1 to 60 starting with the meridian of 180°, the areas of 4° latitude are numbered with big letters of the Latin alphabet from A to V starting with Equator.

The map at scale of 1:1 000 000 was taken as base for the maps of scale 1:250000. It results that for obtaining a map sheet at scale of 1 : 250 000 the map sheet of 1:1 000 000 was divided from 1° to 1° on latitude and from 2° to 2° on longitude that leads to obtaining 12 map sheets at scale of 1:250 000 noted with Arabian numbers from 1 to 12.

- **Map sheet nomenclature at scale of 1:50 000**

Map sheet nomenclature at scale of 1:50 000 is based on the map at scale of 1:1 000 000. The dimensions of the sheet in Romania are 15° on latitude and 18° on longitude. For obtaining a map sheet at scale of 1:50 000 000 the map sheet of 1:100 000 is divided in 4 from 30° to 30° latitude and from 36° to 36° on longitude. The nomenclature of the map sheet is composed of groups of alpha-numerical characters separated by the symbol "x".

The first group of characters is composed of a letter and three figures having the following significance:

- **the first character** (letter) means the region of shore delimited by the NATO interest (Central Europe and South – East Europe). Romania belongs to the M region.

- **the second character** is a figure which shows that the map has a certain scale. This character has a value between 0 – 9 depending on the scale with the following values:

1 – for the scales less than 1: 5 000 000
2 – for scales between 1 : 2 000 000 + 1:5 000 000
3 - for scales between 1: 510 000 + 1:2 000 000
4 - for scales between 1: 255 000 +1:510 000
5 - for scales between 1:150 000 * 1:255 000
6 - for scales between 1; 70 000 + 1: 150 000
7 - for scales between 1: 35 000 ^ 1: 70 000
8 – for scale bigger than 1 :35 000 excepting the town plans
9 – town plans
10-photo maps

- **the third character** is a geographical area from the given region. For Romania, Greece, Bulgaria and ex Yugoslavia is the figure 0.

- **the forth character** is a given sub-area. For Romania is the figure 5.

The second group of characters is formed of 3 groups of figures:

- the first group is formed of 2 figures representing the numbering of the areas of 36° on longitude from east to west.

- the second group is formed of 2 figures representing the numbering of the areas of 30° on latitude from south to north

The third group of characters is formed from a figure which represents the position of the map sheet inside the quadrilateral.

e) The deformations in UTM projection

The relative linear deformation has the following formulas**:**

$$D_{UTM} = k(D_{Gauss} + 1) - 1 = k(L^2 / 2R^2 + L^4 / 24R^4 + 1) - 1 \text{ [km/km]},$$

where:

- o D_{UTM} relative linear deformation in UTM projection;
- o D_{Gauss} relative linear deformation in Gauss projection;
- o R is the average ray of flexion into the considered point;
- o $y=(y-y_0)$ is distance between the point and axial meridian;
- o k is the constant report between the distances from the UTM projection plan and the ones from the Gauss projection plan.

By using this formula for the relative linear deformation in UTM projection there are obtained values which are directly proportional with the distance against the axial meridian and they increase starting with the negative value -40 cm/km (Fig. 6)

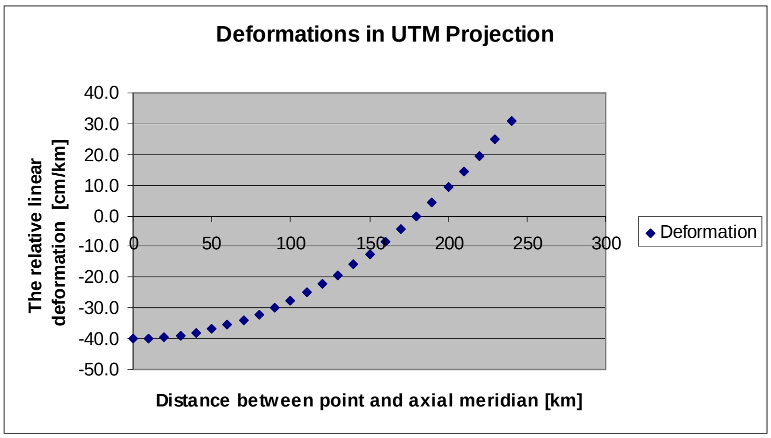

Fig. 6 – The diagram of relative linear deformation in UTM projection

3. Comparisons between the relative linear deformation in UTM and Gauss projections (cm / km)

Reducing the scale in UTM projection make a change of the value and of repartition of the deformations in plan. Each fuse of 6^0 of UTM projection there are 2 lines of null deformations symmetrical against the axial meridian and almost parallel with it at almost 180 km distance.

In south of Romania the lines of null deformations cross with the parallel of 44^0 at a difference of longitude of + and – $2^0 15^{'}$, and at the north the parallel of 48^0 at + and - $2^0 25$'against the axial meridian. So the lines of null deformations from each fuse of 6^0 are beyond the fuse limits of 3^0 that lead to conclusion that to use the fuses of 3^0 in UTM project is useless.

From the diagram presented above (Fig. 2 and Fig. 6) it can be noticed that even if in

Gauss projection all deformations are **positively**, in UTM projection the deformations are also **positively and negatively**.

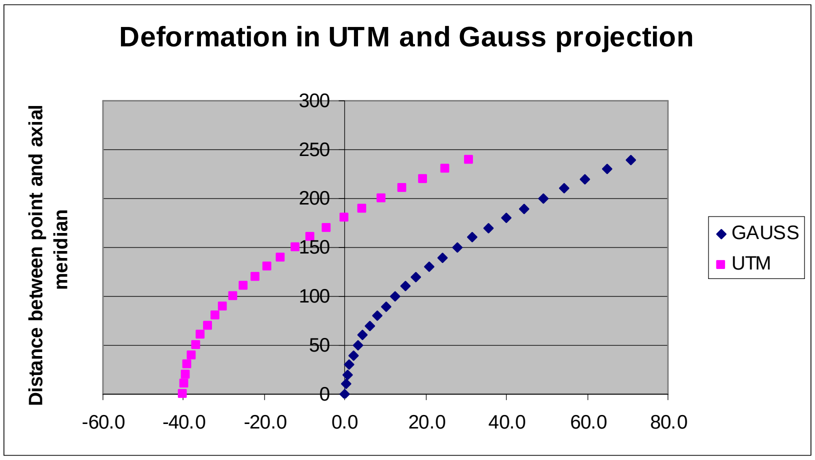

Fig. 7 - Comparative Diagrams of the relative linear deformation in UTM and Gauss projection

CONCLUSION

- The UTM projection refers to whole terrestrial surface;
- The rectangular coordinates system is proper foe each fuse;
- For limiting the deformation errors and obtaining a high precision, the projection is applied on areas delimited by the meridians called fuses;
- Romania is in 2 fuses (L 34, L 35) and the margins of junction of these fuses are of 24° longitude, it results that the adjacent margins are almost in the middle of the country
- The existence of many fuses by in UTM projection it is necessary that at the limit of the 2 fuses it should be the possibility for transforming the coordinates from a fuse to another.

BIBLIOGRAPHY

1. BONEA, I. , DIMA, N. Mathematic Cartography, Lithography of I.M.P., 1968
2. HERBEI, O., Mathematic Cartography – Making and drawing the maps, Eurobit Publishing, Timisoara, 2002
3. BOFU, C., CHIRILĂ, C., Geographic Information Systems. Maps Cartography and edit. Tehnopress Publishing – Iasi. 2007
4. MOCA, V., CHIRILĂ C., - Mathematic Cartography – Making and drawing the maps. Course support. 2002
5. MOROȘANU, B., - Relative linear deformation Stereographic 1970, Gauss -Krüger, UT rojections and comparisons between them, 2007
6. MUNTEANU, C., - Mathematic Cartography, MatrixRom Publishing, București, 2003
7. PALAMARIU, M., PĂDURE, I., ORTELECAN M., - Cartography and chronometry , Aeternitas Publishing, Alba Iulia, 2002