OPTIMISATION OF THE CHEMICAL FERTILIZER DOSE FOR PRODUCTION BENEFIT MAXIMIZATION

M. BOLDEA, F. SALA, F. CRISTA

Banat University of Agricultural Sciences and Veterinary Medicine, Timişoara, Romania
119, CaleaAraduluiStreet, Timişoara,
E-mail: valentinb742002@yahoo.com

Abstract: *The aim of the research in the present paper is to find a mathematical relation between the agricultural yield and the dose of fertilizers, on the one hand; on the other hand, it attempts to give a theoretical and graphic method for benefit optimisation. It is common knowledge that the models used in mathematizing the agricultural yield in relation to the doses of chemical fertilizers with nitrogen, phosphorus and potassium applied on experimental fields are based on non-linear functions. Of these, the ones used most often are exponential functions with negative exponent. Such an example is the one that uses the Mitscherlich function, which is given by relation (1). In the present paper, the function that gives the relation between the agricultural yield,* y *, and the doses of fertilizers,* x *, is given by relation:*

$$y = a\tanh(rx + k),$$

which is a function as good for the case as the ones mentioned above, but different from them in that it uses hyperbolic functions. The constants that are involved in the expression of the above-mentioned function are determined with the least squares method, by comparison with the experimental data. By graphic representation of both function (3) and the experimental data in Table 1, we get the graph in Figure 1. The graph shows good concordance between the theoretic curve and the experimental data. The optimal solution for the dose of fertilizers is obtained by annulling the derivative in the expression of the benefit, or, just as well, by the graphic method given in Figure 2. From the point of view of the practical applications, this paper gives a method for the optimisation of use of the fertilizers with nitrogen, phosphorus and potassium, combining theoretical method with graphic methods. The paper is of practical interest also because it studies the adequate proportions of the three active substances used in fertilizers (N, P, K *); these chemical components are never used in equal percentages, whichever the crop might be.*

Key words**:** *fertilization, Mitscherlich, least squares, modeling*

INTRODUCTION

The interest of any agricultural enterprise is to obtain pecuniary profit from the yield. One of the means used for that purpose is the application of chemical fertilizers. As these fertilizers cost money, it is important that they be used judiciously.

The aim of the research in the present paper is to find a mathematical relation between the agricultural yield and the dose of fertilizers, on the one hand; on the other hand, it attempts to give a theoretical and graphic method for benefit optimisation.

It is common knowledge that the models used in mathematizing the agricultural yield in relation to the doses of chemical fertilizers with nitrogen, phosphorus and potassium applied on experimental fields are based on non-linear functions. Of these, the ones used most often are exponential functions with negative exponent. Such an example is the one that uses the Mitscherlich function, which is given by relation (1).

THE MATHEMATICAL MODEL

The best function that shows the relation between the dose of monofactorial fertilizers and yield is the one given by Mitscherlich:

$$y = a\left(1 - ke^{-bx}\right), \tag{1}$$

obtained as the solution of the following differential equation:

$$\frac{dy}{dx} = c(a - y), \tag{2}$$

that is, the yield increase is proportional to the saturation deficit.

Another function, just as good as the previous one, is:

$$y = a\tanh(rx + k), \tag{3}$$

obtained as the solution of the differential equation:

$$\frac{dy}{dx} = C\left(a^2 - y^2\right), \tag{4}$$

that is, the yield increase is proportional to the saturation super-deficit.

In order to find the values of the constants in relation (3), we use the method of least squares in comparison with the experimental data in table 1.

Table 1.

Experimental data regarding the production of maize, at Timișoara Didactic Station, 2006 - 2008

$x = N + PK$	0	100	200	300	400
$N_{50\%}$	0	50	100	150	200
$PK_{50\%}$	0	50	100	150	200
Production	4585	6049	7425	8160	8865

By using numerical methods, we get the following values for the constants:

$a = 9600, r = 0{,}0026196, k = 0{,}51.$

The graphic representation of both function (3) and the experimental data presented in Table 1 is the one shown in Figure 1.

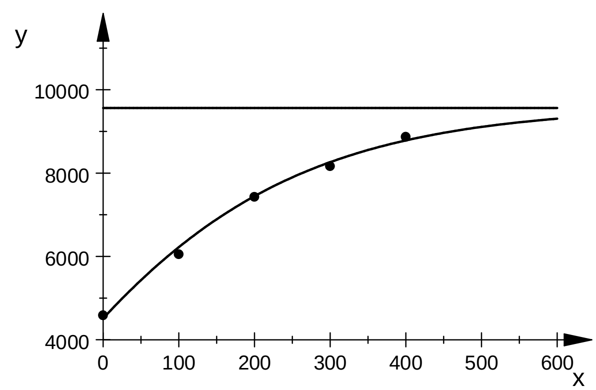

Figure 1.Representation of function (3) and of the experimental data in Table 1

This graph clearly shows good concordance between the theoretical curve given by function (3) and the experimental data.

When looking at the graph, one can also notice the fact that, although the production is always growing, in relation with the doses of fertilizers, the increase is smaller and smaller, in such a way that at a certain point it is no longer profitable to buy the fertilizers for such a small yield increase.

In order to find the optimal dose, we introduce the notion of benefit B , or profit, given by relation:

$$B = V - C \ , \tag{5}$$

meaning the difference between the production value, V , and the cost of fertilizers, C .

Taking into account the market price of the production, $q = 1\frac{RON}{kg}$, and the cost of the fertilizers, $p = 4.455\frac{RON}{kg}$, the benefit becomes:

$$B = aq\tanh(rx + k) - px. \tag{6}$$

But, B is maximum when the derivative is $\left(\frac{dB}{dx} = 0\right)$. Then we get:

$$\frac{aqr}{\cosh^2(rx + k)} - p = 0 \ . \tag{7}$$

If we consider the values of the constants above as an example, then the solution of equation (7) $x_0 = 382$ is the optimal solution for maximum benefit.

Graphically, the optimal solution is obtained by representing V and C (Figure 2):

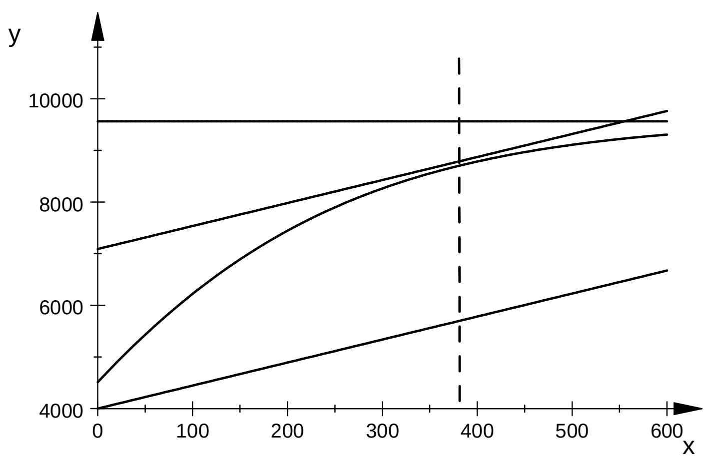

Figure 2. Graphic determination of the optimal solution

where $V = aq\tanh(rx + k)$ and $C = px$.

The optimal solution x_0 is the abscise of the tangent to the curve, parallel with the straight line given by C , that is $x_0 = 382$. It is clear to see that the optimal solution obtained graphically coincides with the solution to equation (7).

CONCLUSIONS

Figure 1 shows good concordance between the graphic and the experimental data.

From the point of view of the practical applications, this paper gives a method for the optimisation of use of the fertilizers with nitrogen, phosphorus and potassium, combining theoretical method with graphic methods. The paper is of practical interest also because it studies the adequate proportions of the three active substances used in fertilizers (N, P, K); these chemical components are never used in equal percentages, whichever the crop might be.

The optimal solution for the fertilizers can be obtained either by annulling the derivative in the expression of the benefit, or by graphic representation, as in Fig. 2

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