ELEMENTS OF TECHNICAL AND ECONOMIC EFFICIENCY WITH NITROGEN FERTILISATION IN WINTER WHET

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Abstract. *To calculate economically and technically optimal fertiliser rates, we frequently use the Mitscherlich-Baule function as a response of the crop to increasing fertiliser rates. Fertilisation has different thresholds of efficiency depending on a number of factors: in agricultural practice, there are situations in which, for economic or technical considerations, we cannot reach optimal fertiliser rates and fertilisation is sometimes even deficitary. Worldwide and in Romania, they are carrying on research in the field of optimising plant nutrition correlated with specific technical and economic factors. Relatively recently, there have also been researches aiming at establishing economically minimal fertiliser rates. Within the research we have carried out, we have taken into account the impact of the factor nitrogen on yield and on some technical and economic optimal elements in winter wheat. This approach is motivated by the fact that they frequently fertilise based on a single nutritious element – nitrogen – as a result of financial constraints. Previous research has allowed us to design a mathematical model for the establishment of the share of microelements of fertilisation of different crops. Unilateral nitrogen fertilisation is justified scientifically and efficient up to a certain level of consumption of nutritious elements as a specific plant nutrition requirement. Using as a mathematical instrument the monofactorial function (2^{nd} degree parabolic function) we have assessed the interdependence between fertiliser rates and yield, the correlation coefficient, maximum fertiliser rate technically and economically, maximum production, and optimal production. We have also assessed a few economic elements generated by the use of fertilisers, cost increase, income increase, benefit increase, as well as maximum benefit. With the fertiliser price of 0.8 Euro/kg of active substance nitrogen (present price), nitrogen fertilisation is profitable up to fertiliser rates of 65-70 kg/ha, a fertiliser rate at which the benefit is annulated by the cost increase. This level of fertilisation can be met frequently on many an agricultural exploitation. With the technical and economic conditions in which we have processed experimental data, optimal yield is 4,010.65 kg/ha that corresponds to a fertiliser rate of 151.24 kg of active substance nitrogen, while the maximum fertiliser rate is of 186.09 kg/ha of active substance nitrogen.*

Key words: *nitrogen fertiliser, economic aspects, optimal fertiliser rates, economic optimum, provisional model*

INTRODUCTION

Plant nutrition is a complex physic-chemical and biological process that is optimal when all nutritious elements are ensured depending on plant requirements. The role of nutrients differs: the share of nitrogen is significantly higher compared to that of other microelements (EPSTEIN & BLOOM 2005).

Unilateral nitrogen fertilisation is justified scientifically and efficient up to a certain consumption level of nutrients as plant nutrition specific requirement (SALA et al. 2007, BOLDEA & SALA 2010). Such fertilisation is achieved from economic reasons as a result of relatively low costs within a technological frame.

Applying fertilisers in increasing amounts leads to lower and lower yields that eventually reach the value 0 if fertiliser and fertiliser application costs keep constant, with a trend increasing proportionally with the fertiliser rate applied (BUDOI 2008, FINGER &

HEDIGER 2008).

Fertilisation has different thresholds of efficiency that depend on a number of factors: in practice, there are both sustainable agricultural systems and situations in which, for economic or technical reasons, we cannot reach optimal fertiliser rates and fertilisation is sometimes even deficitary (HERA et al. 2001, DUMITRU 2002).

To assess the relationship fertiliser – yield both technically and economically, we can use a number of mathematical models that capture and express the level of interdependence between variables (MITSCHERLICH-BAULE, BORLAN & HERA 1982, MAKOWSKI 1999, BUDOI 2004, 2008 FINGER & HEDIGER 2008, AIZPURUA 2010).

Based on research, MITSCHERLICH expressed the dependence of yield increase on the increase factor added through the equation known as the law of vegetation factor action:

$$dy/dx = k(A-y)$$

In our study, we have taken into account the impact of the nutrition factor nitrogen on yield and on some technical and economic optimum in winter wheat.

Using as a mathematical analysis instrument the monofactorial function (2nd degree parabolic function), we have assessed the interdependence between fertiliser rates and yield, correlation coefficient, technical and economic maximum fertilisation rate, maximum yield, and optimal yield. We have also assessed a few economic elements generated by the use of fertilisers: cost increase, income increase, benefit increase, as well as maximum benefit.

MATERIAL AND METHODS

We have assessed the impact of the nutrition factor nitrogen on yield and of some economic elements in winter wheat: experimental variants, mathematical working instruments, and parameters monitored are shown in Figure 1.

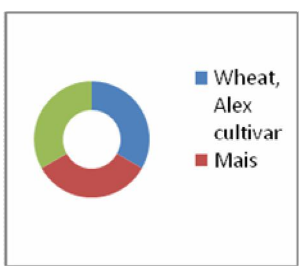	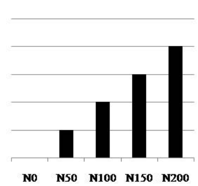 Nitrous fertilization	2nd degree parabolic function $y = A_0 + A_1x \pm A_2x^2$	Values measured: Maximum fertiliser rate Optimal fertiliser rate Maximum yield Optimal yield Cost increase Income increase Benefit increase Maximum benefit
Crop rotation and biological material	Fertilization	Mathematical model	Determined technical and economic elements

Fig. 1. Material and method components

The biological material has been the winter wheat cultivar Alex (*Triticum aestivum* ssp. *vulgare*) within a crop rotation over 3 years – wheat, maize, and sunflower.

We have monitored the impact of the nutrition factor nitrogen on yield and on some economic elements.

As mathematical instrument (model) of analysis of the results, we have used the 2nd degree parabolic function that ensures a high degree of assessment of the relation fertiliser – yield.

We have measured the following elements: maximum fertiliser rate – maximum yield, optimal fertiliser rate – optimal yield, cost increase, income increase, benefit increase, and maximum benefit.

The degree of correlation between the independent variable (fertiliser rate) and the dependent variable (yield and economic elements) was underlined by the correlation coefficient.

RESULTS AND DISCUSSIONS

Expressing the interdependence between the factor nitrogen and yield has been done through a mathematical model in which fertiliser rates (noted by x) are considered the independent variable and yield (noted by y) is considered the dependent variable.

This can be expressed as follows:

$$y = f(x) \quad (1)$$

where:
y = yield (kg/ha);
x = N, (nitrogen fertiliser rates in kg of active substance/ha).

The relationship between the dependent variable (yield) and the independent variable (fertilisation rates) can be expressed through several types of functions (linear, parabolic, exponential, etc.).

In general, there is no proportional increase of the yield depending on the increase of the fertiliser rate. The yield increase due to the increase of the fertiliser rate is lower and lower down to a certain level from which yield diminution is determined by increased fertiliser rates.

We have expressed such a relationship between fertilisers and yield through a 2[nd] degree parabolic function of the following general form:

$$y = A_0 + A_1 x \pm A_2 x^2 \quad (2)$$

where:
y = yield (kg/ha);
A_0 = yield with no fertiliser applied;
A_1 = yield increase as a result of applying one unit of fertiliser;
x = fertiliser rate (kg of active substance/ha);
A_2 = the coefficient of the function of linear regression showing the increase or the decrease of the yield with the square of the nitrogen fertiliser rates.

Table 1 presents experimental data and measurement coefficients.

Table 1

Experimental data and measurement coefficients

Nr.	Yield (kg/ha) y	Nitrogen rate (kg s.a./ha) x	Measurement coefficients
1	2155,75	0	x^2 x^3 x^4 xy x^2y y^2
2	2818,25	50	
3	3577,75	100	
4	4278,25	150	
5	3945,75	200	

In the processing of the experimental data, we have started from the calculus of the elements x^2, x^3, x^4, xy, x^2y, y^2 and of the values A, B, C, D and E with the proper formulas.

$$A = \sum x^2 - \frac{(\Sigma x)^2}{\Sigma y} \qquad (3)$$

$$B = \sum x^3 - \frac{\Sigma x \cdot \Sigma x^2}{n} \qquad (4)$$

$$C = \sum x^4 - \frac{(\Sigma x^2)^2}{n} \qquad (5)$$

$$D = \sum xy - \frac{\Sigma x \cdot \Sigma y}{n} \qquad (6)$$

$$E = \sum x^2 y - \frac{\Sigma x^2 \cdot \Sigma y}{n} \qquad (7)$$

The coefficients of the regression factors have been calculated with the formulas:

$$A_1 = \frac{C \cdot D - B \cdot E}{A \cdot C - B^2} = 21.788 \qquad (8)$$

$$A_2 = \frac{A \cdot E - B \cdot D}{A \cdot C - B^2} = -0.058 \qquad (9)$$

$$A_0 = \frac{\Sigma y}{n} - A_1 \frac{\Sigma z}{n} - A_2 \frac{(\Sigma x)^2}{n} = 2054.43 \qquad (10)$$

A_0 – the coefficient of the regression function, that shows the level of the yield in the variant not fertilised (kg/ha),

A_1 - the coefficient of the regression function, that shows the increase or the decrease of the yield after nitrogen fertiliser application,

A_2 - the coefficient of the regression function that shows the increase or the decrease of the yield with the square of the nitrogen fertilizer rates.

We have assessed the correlation of the yield (dependent variable) with fertiliser rates (independent variables) through the correlation of the two variables (η) according to the relation (11). The theoretical values for the thresholds 5% and 1% are $\eta_{5\%}$ = 0,433 and $\eta_{1\%}$ = 0,549. Since the calculated value is 0.971, we can deduce the significantly distinct relationship between the two variables.

$$\eta = \sqrt{\frac{A_0 \Sigma y + A_1 \Sigma xy + A_2 \Sigma x^2 y - \frac{(\Sigma y)^2}{n}}{\Sigma y^2 - \frac{(\Sigma y)^2}{n}}} = 0.971 \qquad (11)$$

The regression function is obtained by replacing the coefficients A_0, A_1, A_2 in the basic formula of the 2[nd] degree parabolic function. Attributing values for x from 0 to 200, we can get the response values of the yield depending on the fertiliser rates as shown in Table 2 and Figure 2.

$$y = A_0 + A_1 x + A_2 x^2 \qquad (12)$$

We have calculated maximum fertiliser rate represented by the amount of fertilisers resulting in maximum yield through derivation starting from the monofactorial function (relations (13), (14), and (15)). We have measured maximum yield by replacing in the yield function (12) the value of the maximum fertiliser rate (16).

$$y = A_1 0 + A_1 1\, x + A_1 2\, x^\dagger 2 \quad ((\qquad (13)$$

$$y^\dagger (= 2A_1 2\, x + A_1 1 \qquad (14)$$

$$x_{max} = \frac{A_1}{2A_2} = 186.09\ kg\ N\ s.a. \frac{\square}{ha} \qquad (15)$$

$$y_1 max = A_1 0 + A_1 1\ x_1 max + A_1 2\ x_1 \mathbf{max}^\dagger 2 = 4081.76\ kg/ha \qquad (16)$$

Applying fertilisers results in an increase of the costs per area unit (ha) or per yield (kg or t).

The function of the cost increase takes into account the fertiliser cost (C_i), the cost of applying the fertiliser (C_{ai}), and the cost of harvesting and of transporting the yield surplus (C_{rt}) as a result of fertilisation, rendered in the relation (17).

$$\Delta c = C_{rt}(A_1 x + A_2 x^2) + C_i \cdot x + C_{ai} \cdot x$$
(17)

where:
C_{rt} – cost of harvesting and of transporting the yield
C_i – cost of nitrogen fertiliser
C_{ai} – cost of applying the fertiliser

The function of the value increase takes into account the yield and the price of valorisation according to the expression (18), the values thus obtained are rendered in Table 2 and in Figure 2.

$$\Delta v = \mathrm{Pr}(A_1 x + A_2 x^2) \qquad (18)$$

The benefit is obtained through the difference between the value of the yield and the costs of production, according to the relation (19).

$$\Delta b = \Delta v - \Delta c \qquad (19)$$

Attributing values for x in the relations (17), (18), and (19), we get concrete values for $\Delta c, \Delta v, \Delta b$ as shown in Table 2 and Figure 2.

With the fertiliser price of 0.8 Euro/kg of active substance nitrogen (present price), nitrogen fertilisation is profitable up to fertiliser rates of 65-70 kg/ha, a fertiliser rate at which the benefit is annulated by the cost increase. This level of fertilisation can be met frequently on many an agricultural exploitation.

If the subvention covers part of the costs of fertilisers, fertilisation profitability increases: thus, for a fertiliser price of 0.6 Euro/kg of active substance, the benefit of fertilisation is annulled for a fertiliser rate of 138 kg of active substance, while at a fertiliser price of 0.5 Euro/kg fertilisation profitability is up to a fertiliser rate of 175 kg of active substance/ha.

From an economic point of view, optimal fertiliser rate is the amount of fertiliser resulting in maximum benefit. Mathematically, optimal fertiliser rate is obtained by equalling the derivate of the function of the value increase Δv with the derivate of the function of the cost increase Δc and by solving the equity, relation (20). We measured optimal yield on the ground of the optimal fertiliser rate, relation (21).

$$d\boldsymbol{\Delta}v = d\boldsymbol{\Delta}c \ ((\tag{20}$$

$$x_{opt} = 151{,}0924\ kg\ N\ s.a.\frac{\square}{ha}$$

$$y_{opt} = A_0 + A_1 x_{opt} + A_2 x_{opt}^2 = 4010{,}65\frac{kg}{ha} \tag{21}$$

Maximum benefit increase has been obtained by subtracting the cost increase from the value increase, calculated for the optimal fertiliser rate, relation (22).

$$b_{max} = v_{opt} - c_{opt} = 147{,}18\ Euro \tag{22}$$

The values obtained for all the parameters measured are marked on the yield curves and on the value curves shown in Figure 2.

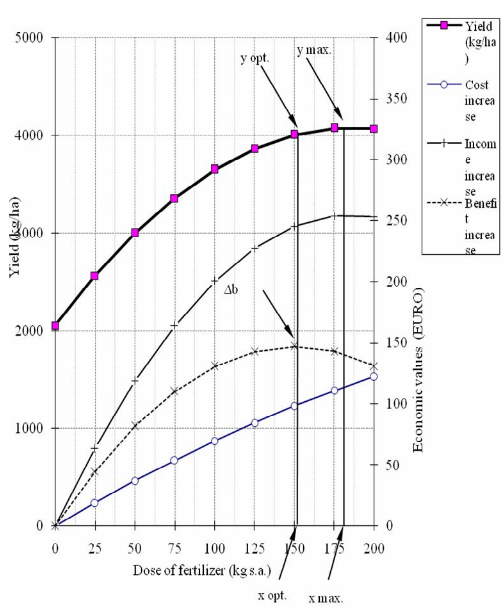

Figure.2. Diagramme of the yield curve and of economic elements as a response to the nitrogen nutrition factor

Table 2

Values of the parameters measured on the ground of the mathematical model (2nd degree parabolic function) under the impact of the factor nitrogen in winter wheat

x	Y	Cost increase Δc	Income increase Δv	Benefit increase Δb
0	2054,43	0	0	0
50	2997,50	37,028	63,92	44,87
100	3647,86	69,781	200,45	130,67
150	4005,50	98,256	245,44	147,17
151,24*	4010,65	97,78	246,09	147,19
186,09**	4081,76	116,15	255,03	138,88
200	4070,43	122,455	253,61	131,15

* optimal values

** maximum values

CONCLUSIONS

Applying fertilisers in increasing amounts leads to lower and lower yields that eventually reach a certain value if fertiliser and fertiliser application costs keep constant, with a trend increasing proportionally with the fertiliser rate applied.

Fertilisation has different thresholds of efficiency depending particularly on the economic factor (costs incumbent with fertilisation). In agricultural practice, there are situations in which, because the costs of the fertilisers, fertilisation level is low, much below the technically or economically optimal rate.

The 2^{nd} degree parabolic function is the mathematical instrument reflecting very accurately the relationship between fertilisers and yield.

With the fertiliser price of 0.8 Euro/kg of active substance nitrogen (present price), nitrogen fertilisation is profitable up to fertiliser rates of 65-70 kg/ha, a level of fertilisation can be met frequently on many an agricultural exploitation.

If the subvention covers part of the costs of fertilisers, fertilisation profitability increases: thus, for a fertiliser price of 0.6 Euro/kg of active substance, the benefit of fertilisation is annulled for a fertiliser rate of 138 kg of active substance, while at a fertiliser price of 0.5 Euro/kg fertilisation profitability is up to a fertiliser rate of 175 kg of active substance/ha.

With the technical and economic factor conditions in which we have processed experimental data, optimal yield is 4,010.65 kg/ha corresponding to an optimal fertiliser rate of 151.24 kg active substance N/ha while the maximum fertiliser rate of 186.09 kg active substance N/ha ensures a yield of 4,081.76 Kg/ha.

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