OPPORTUNITIES OF SIMULATION FOR STANDS STRUCTURE USING MATHEMATICAL MODELS

A.M. BICA[1], G.C. CRAINIC[2], M. CURILĂ[2], S. CURILĂ[3]
[1] University of Oradea, Faculty of Sciences,
[2] University of Oradea, Faculty of Environmental Protection,
[3] University of Oradea, Faculty of Electrical Engineering and Information Technology
e-mail:abica@uoradea.ro, gccrainic@yahoo.com,

Abstract. *The analysis, study and simulation of pure and/or mixed stand structure represent a particularly important problem in the forest management planning process, and respectively in the sustainable potential forest wood's administration, management and exploitation.The stand's structure characterizes the composition, organization and functioning preferably of the tree storey with major implications on other parts of the forestry phytocoenosis and thus on forest ecosystems overall.Silvotechnics interventions are directly related to the stands structure, as a result the complex stand structure - silvotechnics intervention-forestry strategy is defining for the integrated forest management, related to the national forest stock, regardless of the type of ownership and administration.Currently there are possibilities to study, analyze and simulate various types of stands structure, using for this purpose the mathematical modelling, which in the present paper is based on splines and algorithms, for whose implementation we used MATLAB programming environment. In the conducted case study it was examined and analyzed the structure of a stand mixture consisting of spruce and beech species, using mathematical models with cubic splines interpolating the experimental data and using the interpolation results for simulating the corresponding normal probability distribution function. The results led to the conclusions and relevant recommendations to current practical activities (needed silvotechnics interventions) done in these stands.*

Keywords*: stand, pure stand, mixed stand, stand structure, simulation of stand structure, experimental distribution, theoretical distribution.*

INTRODUCTION

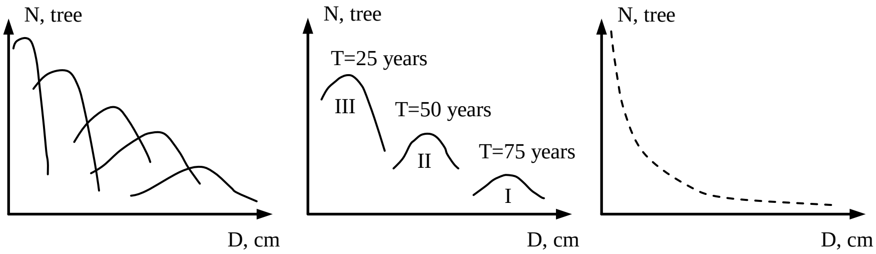
The stands structure is a very important feature in their conducting process, being expressed by a number of elements horizontally and vertically of a qualitative and synthesis nature.

As a result, the structure of the stand represents the expression of the constituent elements related to the tree storey in the forestry phytocoenosis, according to their qualitative and quantitative characteristics (species, base diameter, number, quality class, etc.) (Crainic G.C., 2016).

Frequently, the distribution number of trees in the diameter categories - Figure 1.1. can provide a range of information on the stand's structure and thus on its functional structural peculiarities, with specific implications for silvotechnics interventions needed to be applied on the stand.

The distribution of trees on diameter categories has a total of features, depending on the used data. As a result, for the recorded data on the ground by full or partial inventories (statistical and mathematical) the experimental distributions are obtained and implicitly, the corresponding theoretical distributions, based on the used algorithms in direct correlation with the objectives.

The mathematical model involved in our study is represented by a set of cubic splines applied for interpolating both absolute and relative frequencies of the observed diameters for pine spruce, beech and for the whole stand.



a).even-aged stands of different ages b).stands for multiple ages c). plurien stands

Figure 1.1.- Distribution (variation) of the trees number of diameter category of the different types of stands (Florescu I.I., Nicolescu N. V., 1996)

The histogram of relative frequencies is associated with a theoretical normal distribution function for which the corresponding parameters mean and standard deviation are determined by using the experimental mean and variance values correlated with the Pearson's skewness coefficient.

MATERIAL AND METHODS

The case study was performed in a stand with a mixed composition, consisting of spruce - *Picea abies* L., H. Karst. and beech - *Fagus sylvatica* L. . The number of trees was counted for each diameter (between 10 and 42 centimeters).

Table 1

Input data

D (cm)	Spruce (*Picea abies*) species		Beech (*Fagus sylvatica*) species		Stand	
	Abs. Freq. (N/ha)	Rel. Freq.	Abs. Freq. (N/ha)	Rel. Freq.	Abs. Freq. (N/ha)	Rel. Freq.
0	*1*	*2*	*3*	*4*	*5*	*6*
10	100	0,087489	45	0,080071	145	0,085044
12	148	0,129484	90	0,160142	238	0,139589
14	176	0,153981	136	0,241993	312	0,182991
16	200	0,174978	92	0,163701	292	0,171261
18	145	0,126859	87	0,154804	232	0,136070
20	112	0,097988	43	0,076512	155	0,090909
22	65	0,056868	21	0,037367	86	0,050440
24	60	0,052493	11	0,019573	71	0,041642
26	30	0,026247	10	0,017794	40	0,023460
28	30	0,026247	8	0,014235	38	0,022287
30	15	0,013123	9	0,016014	24	0,014076
32	10	0,008749	3	0,005338	13	0,007625
34	11	0,009624	2	0,003559	13	0,007625
36	10	0,008749	1	0,001779	11	0,006452
38	12	0,010499	2	0,003559	14	0,008211
40	10	0,008749	1	0,001779	11	0,006452
42	9	0,007874	1	0,001779	10	0,005865
Total	**1143**	-	**562**	-	**1705**	-

The total number of the spruces species is 1143 and of the beech species is 562 trees. The amount of all trees (stand) have been counted, being 1705. The measurement results are integrated in *Table 1*, where we have included both absolute frequency and relative frequency for each diameter.

Considering the diameter on the abscises axis *Ox*, with x_i in the interval $[D_0, D_n] = [10,42], i = \overline{0,n}, n = 16$, and the absolute frequency $y_i, i = \overline{0,n}$, on the ordinates axis *Oy*, we construct three types of cubic splines for interpolating the points $(x_i, y_i), i = \overline{0,16}$. In order to simulate the empirical statistic distribution of each species and of the *brush* we consider $\overline{y} = \sum_{i=0}^{n} y_i$ and the relative frequencies $\overline{y_i} = \frac{y_i}{y}, i = 0,16$. In that follows we present the mathematical method of spline interpolation applied in our study to the data from *Table 1*.

The involved cubic splines will be the C^2-smooth natural cubic spline (Ahlberg 1967, pages 12-13), the Akima's cubic spline (Akima 1970) and its optimized version near end-points (Bica 2014). Both these splines have expression using the cubic Hermite basis functions on each subinterval. With $h_i = x_i - x_{i-1}, i = \overline{1,n}$, be the stepsize of the mesh, the Hermite cubic polynomials on each subinterval $[x_{i-1}, x_i], i = \overline{1,n}$ are:

$$s(x) = \frac{(x_i - x)^2 [2(x - x_{i-1}) + h_i]}{h_i^3} \cdot y_{i-1} + \frac{(x - x_{i-1})^2 [2(x_i - x) + h_i]}{h_i^3} \cdot y_i + \frac{(x_i - x)^2 (x - x_{i-1})}{h_i^2} \cdot m_{i-1} - \frac{(x - x_{i-1})^2 (x_i - x)}{h_i^2} \cdot m_i \tag{1}$$

where the derivatives $m_i = s'(x_i), i = \overline{0,n}$, are computed by using special procedures established for each type of splines. Generally, the obtained cubic spline *s* is smooth, with continuous first order derivative, excepting the natural cubic spline for which we have continuous second derivative.

The values of $m_i, i = \overline{0,n}$, are determined for the natural cubic spline using the smoothness condition (*s*'' continuous on [x_0, x_n]) and the natural end-point conditions $s''(x_0) = s''(x_n) = 0$. These lead to the following three-diagonal linear system, diagonally dominant:

$$2m_0 + m_1 = \frac{3(y_1 - y_0)}{h_1}, \quad m_{n-1} + 2m_n = \frac{3(y_n - y_{n-1})}{h_n}$$

$$\frac{1}{h_i} \cdot m_{i-1} + 2\left(\frac{1}{h_i} + \frac{1}{h_{i+1}}\right) \cdot m_i + \frac{1}{h_{i+1}} \cdot m_{i+1} = \frac{3(y_i - y_{i-1})}{h_i^2} + \frac{3(y_{i+1} - y_i)}{h_{i+1}^2}, \; i = \overline{1,n-1} \tag{2}$$

which has unique solution and it can be exactly solved by applying the iterative algorithm presented in Ahlberg 1967, pages 14-15.

The values of $m_i, i=\overline{0,n}$, for the Akima's cubic spline are obtained as follows. Firstly, are computed the slopes $p_i = \dfrac{y_{i+1}+y_i}{x_{i+1}+x_i}, i=\overline{0,n-1}$, and for $i=\overline{2,n-2}$ the derivatives are

$$m_i = \frac{|p_{i+1}-p_i|\cdot p_{i-1} + |p_{i-1}-p_{i-2}|\cdot p_i}{|p_{i+1}-p_i| + |p_{i-1}-p_{i-2}|},\ \text{if } |p_{i+1}-p_i| + |p_{i-1}-p_{i-2}| \neq 0, \text{and}$$
$$m_i = \frac{|p_{i-1}+p_i|}{2},\ \text{for} |p_{i+1}-p_i| = |p_{i-1}-p_{i-2}| = 0 \tag{3}$$

Secondly, formula (3) is extended for $i=\overline{0,n}$, by introducing four supplementary slopes

$$p_{-1} = 2\,p_0 - p_1,\ p_{-2} = 3\,p_0 - 2\,p_1,\ p_n = 2\,p_{n-1} - p_{n-2},\ p_{n+1} = 3\,p_{n-1} - 2\,p_{n-2}$$

In order to avoid the artificial construction of the slopes p_{-1}, p_{-2}, p_n, p_{n+1}, that sometimes lead to unjustified swelling near the end-points (Bica 2012, Figure 1 in page 2051), we have proposed (Bica 2014) an optimal alternative by minimizing near end-points, on the first two and on the last two subintervals, the residual

$$R(m_0, m_1, m_{n-1}, m_n) = \sum_{i=1}^{2} \int_{x_{i-1}}^{x_i} \left[s(x) - \frac{x_i - x}{h_i}\cdot y_{i-1} - \frac{x - x_{i-1}}{h_i}\cdot y_i \right]^2 dx +$$
$$+ \sum_{i=n-1}^{n} \int_{x_{i-1}}^{x_i} \left[s(x) - \frac{x_i - x}{h_i}\cdot y_{i-1} - \frac{x - x_{i-1}}{h_i}\cdot y_i \right]^2 dx$$

Taking $m_i, i=\overline{2,n-2}$, as in (3) and solving the normal equations

$$\frac{\partial R}{\partial m_0} = 0,\ \frac{\partial R}{\partial m_1} = 0,\ \frac{\partial R}{\partial m_{n-1}} = 0,\ \frac{\partial R}{\partial m_n} = 0$$

in the case of equidistant knots (this is the cases in our application according Table 1), we obtain

$$m_0 = \frac{9m_2}{23} + \frac{11(y_1 - y_0)}{23h} + \frac{3(y_2 - y_1)}{23h}$$
$$m_1 = \frac{12m_2}{23} + \frac{7(y_1 - y_0)}{23h} + \frac{4(y_2 - y_1)}{23h}$$
$$m_{n-1} = \frac{12m_{n-2}}{23} + \frac{7(y_n - y_{n-1})}{23h} + \frac{4(y_{n-1} - y_{n-2})}{23h}$$
$$m_n = \frac{9m_{n-2}}{23} + \frac{11(y_n - y_{n-1})}{23h} + \frac{3(y_{n-1} - y_{n-2})}{23h}$$

where h is the uniform stepsize.

Now, including in (1) the computed values of $m_i, i=\overline{0,n}$, the splines are completely determined.

Definition 1 (Lorentz 1986): For a function *f*, continuous on [*a*, *b*], its modulus of continuity is

$$\omega(f,d)=\max\{|f(t)-f(s)|:t,s\in[a,b],|t-s|\le d\}$$

for $0\le d\le b-a$.

Under equidistant knots, the error estimate in the interpolation with the above presented splines is:

$$|f(x)-s(x)|\le\frac{7}{4}\omega(f,h),\quad x_0\le x\le x_n \text{, for natural cubic spline,}$$

$$|f(x)-s(x)|\le\frac{5}{4}\omega(f,h),\quad x_1\le x\le x_{n-1}$$

and

$$|f(x)-s(x)|\le\frac{7}{4}\omega(f,h),\quad x\in[x_0,x_1]\cup[x_{n-1},x_n]$$

for the Akima's spline, and

$$|f(x)-s(x)|\le\frac{5}{4}\omega(f,h),\quad x_0\le x\le x_n$$

in the case of the Akima's spline optimized near the end-points, where $h=\max\{h_i : i=\overline{1,n}\}$ and f is a continuous function with $f(x_i)=y_i, i=\overline{0,n}$. Since $\omega(f,h)\to 0$ for $h\to 0$, these estimates ensure the convergence of the spline interpolation procedure.

The above presented splines are applied to interpolate the absolute frequencies $y_i, i=\overline{0,n}$. Investigating the data from Table 1, the relative frequencies $\overline{y_i}, i=\overline{0,n}$ are interpolated by the same three types of cubic splines, too. For practical purposes we can consider the study of the deviation of the spline interpolating the points $(x_i,\overline{y_i}), i=\overline{0,n}$, by a theoretical probability distribution. Therefore, we compute the expected value $m=\sum_{i=0}^{n}x_i\cdot\overline{y_i}$ and the standard deviation $\sigma=\sqrt{\sum_{i=0}^{n}(x_i-m)^2\cdot\overline{y_i}}$, constructing the corresponding theoretical distribution.

Definition 2: For given data $x_i, i=\overline{0,n}$, with $a = x_0 < x_1 < ... < x_{n-1} < x_n = b$, consider the histogram $(x_i, y_i), i=\overline{0,n}$ of relative frequencies and the corresponding continuous probability distribution $f:[a,b]\to[0,1]$. Let s be a continuous function interpolating the data, $s(x_i)=y_i, i=\overline{0,n}$. The measure of local deviation of s by f on a subinterval $[c, d]$ of $[a, b]$ is

$$I(c,d,\varphi;s)=\sqrt{\int_c^d [s(x)-f(x)]^2\,dx}$$

When $f=N(m,\sigma^2)$ is the normal distribution and $[c,d]=[m-\sigma, m+\sigma]$ the value of $I(m-\sigma, m+\sigma, N(m,\sigma^2):s)$ can be called the measure of central deviation of s by $N(m,\sigma^2)$. In our study, considering the corresponding theoretical distribution to the histogram of relative frequencies and the cubic spline interpolating the relative frequencies $\overline{y_i}$ of the diameters $x_i, i=\overline{0,16}$, we will use the measure of local deviation in order to establish the necessary interventions for the management of forest evolution.

RESULTS AND DISCUSSIONS

The mathematical methods of spline interpolations applied in our study to the input data from *Table 1* have been implemented as procedures using the MATLAB programming language. On the basis of the processed data by these applications have been built the graphs from the following graphical windows, made in the MATLAB programming environment, by plotting the vector of output data according to the vector of input data.

The absolute frequencies of pine spruce, beech and stand from *Table 1* are interpolated by using natural cubic splines (NCS), the Akima's cubic spline (ACS), and the optimized version of the Akima's spline near end-points (OAS). The results are presented in Figures 3.1-3.3, where for the graph of NCS we have used red colour, the graph of ACS appears in blue, and the colour of OAS is green.

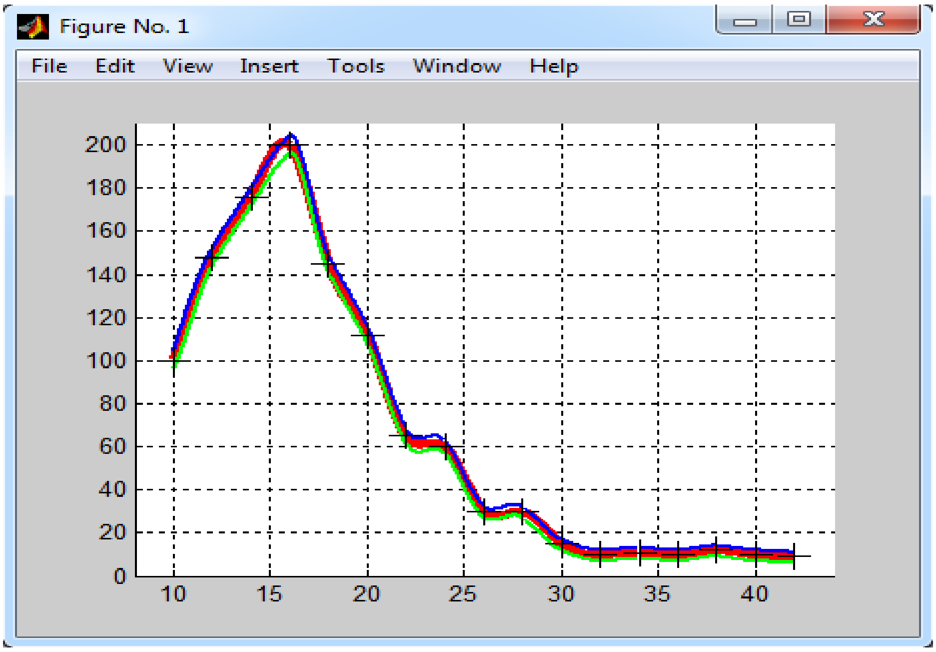


Figure 1. Plotting the absolute frequencies of the spruce (*Picea abies)* species

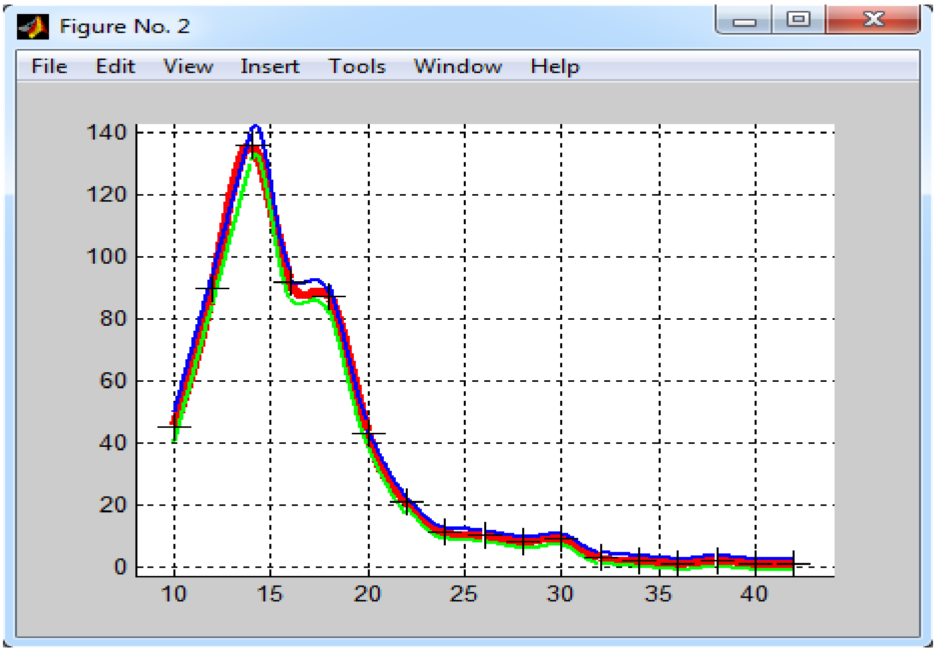


Figure 2. Plotting the absolute frequencies of the beech (*Fagus sylvatica)* species

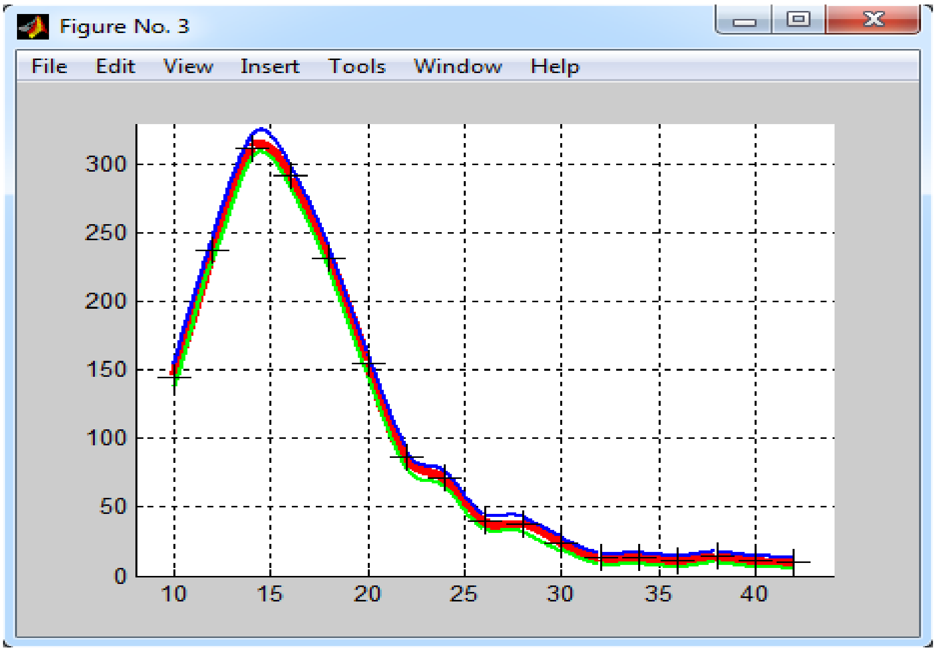


Figure .3. Plotting the absolute frequencies for stand

Regarding the relative frequencies, using the data in *Table 1*, the mode (*Mo*), the expected value (mean) and the standard deviation of the spruce species, beech species, and stand are:

Table 2

Mode / mean / standard deviation

Statistical indicators	**Species**		**Stand**
	Spruce (*Picea abies*)	**Beech (*Fagus sylvatica*)**	
0	***1***	***2***	***3***
Mo	16	14	14
m	17.951	16.31673	17.41232
σ	6.541426	4.990315	6.122462
σ^2	42.79025	24.90324	37.48454

As it can be observed in Figures 3.1-3.3, the histograms have a positive skewness and intending to describe possible interventions that could reduce the asymmetry we associate the corresponding normal distributions. Based on the mode values and considering their relative frequencies we can determine the corresponding standard deviation of the theoretical normal distribution for spruce species, beech species and stand, respectively. Using the empirical mean values from *Table 2* and the Pearson's skewness coefficients we get the mean values of these theoretical distributions. So, the theoretical mean and standard deviation for the spruce species are m_t= 18, σ_t = 2.25 . For the beech species and stand we have obtained m_t = 16.5, σ_t = 1.65 and m_t = 17.5, σ_t=2.15, respectively. Then the probability distribution function for the spruce species and beech species are:

$$f\left(m_t,\sigma_t^2\right)(x)= f(18{,}5.0625)(x)=\frac{1}{2.25\cdot\sqrt{2\pi}}\cdot e^{-\frac{(x-18)^2}{10.125}},\ x\in[10{,}42]$$

and

$$f(16.5{,}2.7225)(x)=\frac{1}{1.65\cdot\sqrt{2\pi}}\cdot e^{-\frac{(x-16.5)^2}{5.445}},\ x\in[10{,}42]$$

respectively.

In Figure 3.4 in the same axes we represent the natural cubic spline (in red) interpolating the relative frequencies $\overline{y_i}, i=\overline{0,16}$ of the spruce species and the normal distribution *f*(18, 5.0625) (in black). Similarly, in Figure 3.5 are represented the relative frequencies $\overline{y_i}, i=\overline{0,16}$ of the beech species interpolated by the optimized version of the Akima's spline near end-points (in green) and the normal distribution *f* (16.5, 2.7225). The relative frequencies $\overline{y_i}, i=\overline{0,16}$ of stand were interpolated using the Akima's cubic spline (in blue) and the normal distribution *f* (17.5, 4.6225) was plotted (in black) in Figure 3.6.

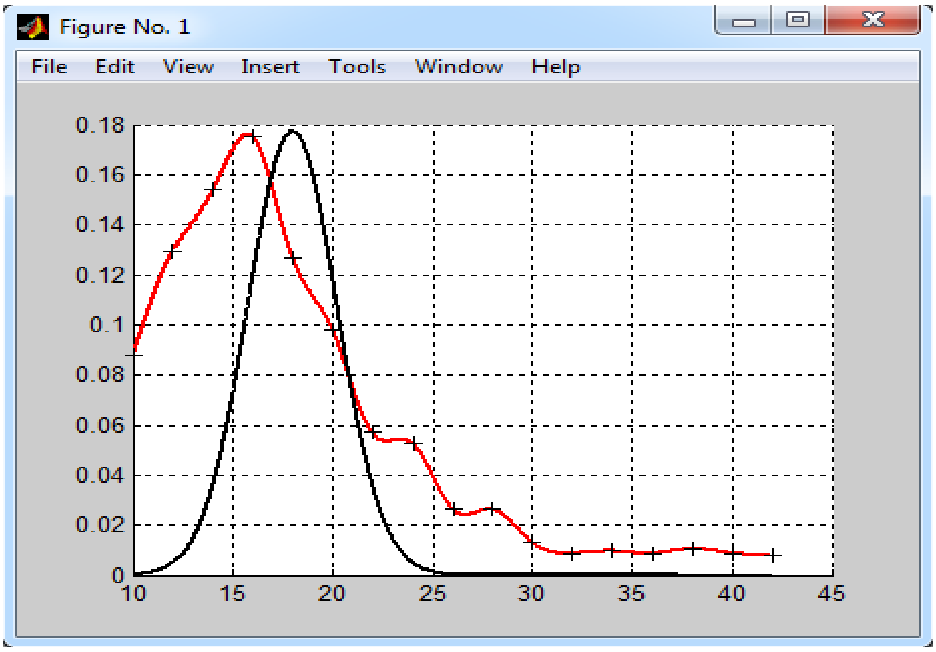


Figure .4. Plotting the relative frequencies for the spruce (*Picea abies)* species

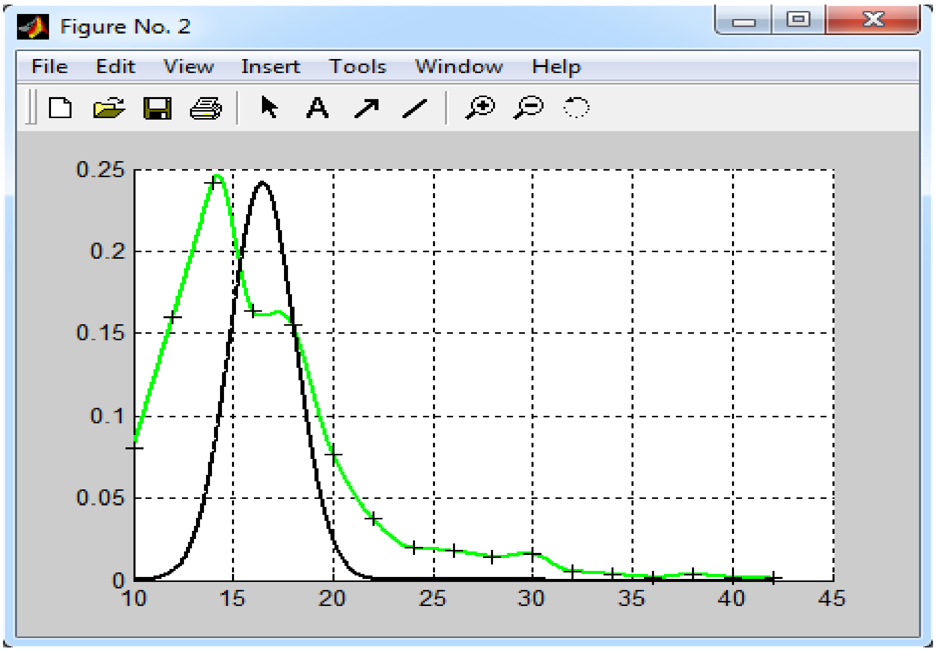


Figure .5. Plotting the relative frequencies for the beech (*Fagus sylvatica)* species

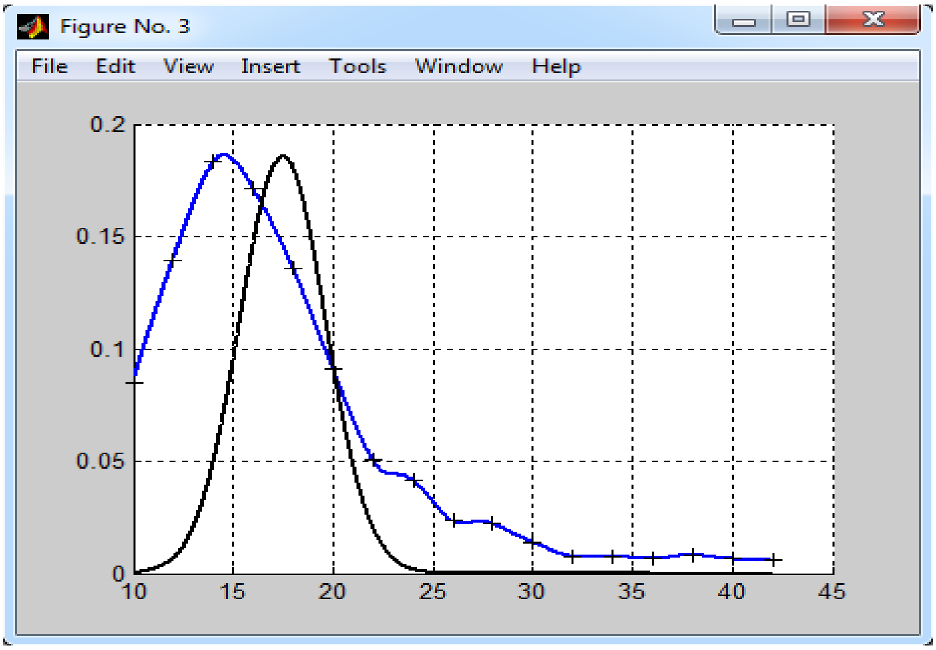


Figure .6. Plotting the relative frequencies for stand

In order to illustrate the use of the measure of local deviation of *s* by the suitable theoretical distribution consider the natural cubic spline interpolating the relative frequencies of pine spruce and compute the measure of central deviation and the measure of local deviation on the interval [17, 21] :

$$I_C\,(15.75, 20.25, f\,(18, 5.0625)\,;\, s) = 0.089938,\; I_L\,(17, 21, f\,(18, 5.0625)\,;\, s) = 0.067834.$$

The measure of local deviation is computed for the beech species on the interval [15, 18] being, $I_L(15, 18, f\,(16.5, 2.7225)\,;\, s) = 0.098425$. The values of the measure of local deviation computed above correspond to a deficit regarding the diameters between 17 cm and 21 cm for the spruce species and between 15 cm and 18 cm for beech, respectively. We see that the relative deficit of the beech species is bigger than those of the spruce species. Regarding the diameter intervals from *Table 1*, we can say that the dominant deficit for the spruce species is in the interval [18, 20], being 0.058226 and similarly, the dominant deficit for the beech species is 0.087571 in the interval [16, 18].

CONCLUSIONS

The obtainment of suitable structures for the various stands made of broad-leaved, resinous and/or mixture species requires the determination of experimental distributions corresponding the trees number on diameter categories, and respectively determining the diameter of splines and the corresponding theoretical distributions.

The applicability of the results will be analyzed and compared to the experimental distributions and to the theoretical ones related to the studied stand in order to establish relevant conclusions and implementation possiblilities in the current practice of effective solutions designed to optimize the production process in the forestry sector.

In the compared analysis of the diagrams from the Figure 3.4, it is found that in the spruce species in the diameter within the range of 10-16 (cm) there is a surplus of trees, in the range 16.1 to 22 (cm) there is a shortage of trees and whithin the range 22.1 -42 (cm) there is a tree surplus.

As a result, for the spruce species is recommended a low thinning of high-intensity for the diameter class of 10 to 16 (cm) and a combined thinning of strong intensity in the diameter class of 22.1 to 42 (cm). For the diameter class of 16.1 to 22 (cm), given the state of the stand component, it is recommended a sanitary cutting, intervention designed to ensure a corresponding phytosanitary state.

By comparatively analysing the diagrams from the Figure 3.5, and by using the above computed values of measure of local deviation and interval with dominant deficit, it can be concluded that the beech species, in the diameter class of 10-16 (cm) there is a surplus of trees, also for the diameter class of 16.1 to 18 (cm) it has been found a deficit of trees, and within the diameter class of 18.1 to 42 (cm) it has been identified a surplus of trees.

In this context, for the beech species it will be necessary a negative selection intervention (low thinning) of high intensity for the trees with a diameter range of 10 to 16 (cm) and a positive selection (combined thinning) with a moderate-intensity for the trees with a diameter range of 18.1 to 42 (cm). From the diameter class of 16.1 to 18 (cm) will be extracted only the affected samples by various pathogens, to provide a suitable phytosanitary of the stand component.

At the stand level, the analysis of distributions from the the Figure 3.6, it can be concluded that the diameter ranges from 10 to 17 (cm) and 20.1 to 42 (cm) it is highlighted a tree overplus, and for the 17.1 to 20.0 (cm) interval the number of trees is deficient.

Silvotechnics interventions which are necessary to be applied in the analyzed and studied stand has a number of features depending on the experimental and theoretical distributions related to the species consisting the stand.

Accordingly, for the diameter range of 10 to 17 (cm) is necessary to perform a negative selection of high intensity, and for the diameter range of 20.1 to 42 (cm) is recommended a positive selection (combination thinning) of moderate - strong intensity. For the diameter range of 17.1 to 20.0 (cm) it is recommended an intervention only to provide a suitable phytosanitary (sanitary cutting).

Based on comparative analysis of the diagrams from the Fiigure 3.4, 3.5 and 3.6 it is found that at the stand level the necessity for interventions differentiated from a specific and intensity point of view, on species and diameter intervals (class).

Using various mathematical models for the presentation of the stand structure it can be determined and thus simulated, at species level (and/or stand component) the specificity and respectively the intensity (amount) of the needed silvotechnics intervention required for a certain stage of development of the stands, for the appropriate stage of development. Also, based on proper analysis, some information can be obtained referring to the periodicity of silvotechnics interventions.

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